Rudimentary Model Atmosphere

Olof van den Berg

27-10-2003

1 The Planck function

1.1 Plotting the Planck function

The first assignment was to plot a graph of the Planck function versus ν, for temperatures 2000, 3000, 4000, 6000, 8000, 10000, 20000 and 30000 K, between 3000 and 10000 $\AA$. The Planck function is:

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5}\left(e^{\frac{ht}{\lambda kT}} - 1\right)^{-1}. \tag{1}$$

To do this, I used Mathematica, in which I used the following commands:

```
B[l_,t_]:=(2*h*c^2)/((Exp[(h*c)/(l*k*t)]-1)*l^5)

h=6.6*10^(-34)
k=1.4*10^(-23)
c=3*10^8

Plot[{Log[10,B[l, 2000]], Log[10,B[l, 3000]], Log[10,B[l, 4000]],
    Log[10,B[l, 6000]], Log[10,B[l, 8000]], Log[10,B[l, 10000]],
    Log[10,B[l, 20000]], Log[10,B[l, 30000]]},
    {l, 2000*10^(-10), 10000*10^(-10)}]
```

This resulted in the following figure:

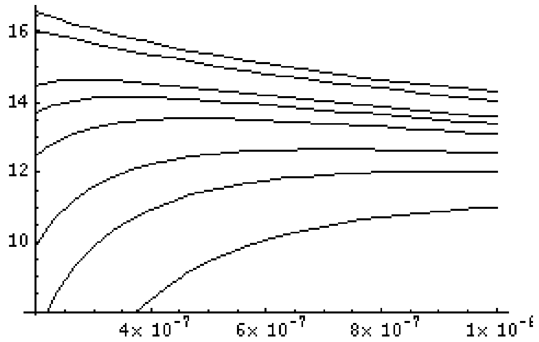

Figure 1: The Planck function for 2000, 3000, 4000, 6000, 8000, 10.000, 20.000 and 30.000 K: Flux Log(J/m^2 s) against the wavelength λ (m).

1.2 Numerical approximation of the flux

For the next part I used Mathematica's NIntegrate function. For different temperatures I calculated the flux with the Planck function and the standard theoretical formula for the flux:

$$F(T) = \frac{\sigma T^4}{\pi} \tag{2}$$

For the σ I used the equation

$$\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2} = 6,057 \times 10^{-8} W m^{-2} K^{-4} \tag{3}$$

In Mathematica I did

```
B[l_, t_] := (2*h*c^2)/((Exp[(h*c)/(l*k*t)] - 1)*l^5)

NIntegrate[B[l, T], {l, 1*10^(-100), 1000000*10^(-10)}]
```

This resulted in the following table:
The values for flux are in $\frac{J}{sm^2}$. The NIntegrate function already takes the most efficient amount of gridpoints to approximate the flux, so you already get the best answer. But you can also set the maximal amount of gridpoints and mmake it lower to make a comparison. Quite obviously does increasing the amount of gridpoints improve the results. For a logarithmic grid this wil go even better, because you can have more gridpoints than in the normal calculation.

1.3 Solar temperatures and spectrum

For the next part of the assigment, the calculation of the effective temperature, I used 3 formulas:

$$L_\odot = S_\odot 4\pi D_\odot^2, \tag{4}$$

Table 1: Theoretical and numerical values of the flux at different temperatures.

T (K)	theoretic	numeric
2000	308.529	308.524
3000	1.56193×10^6	1.56192×10^6
4000	4.93646×10^6	4.93645×10^6
6000	2.49909×10^7	2.49908×10^7
8000	7.89834×10^7	7.89834×10^7
10000	1.92831×10^8	1.92831×10^8
12000	3.99854×10^8	3.99854×10^8

$$R_\odot = D_\odot \tan(0.267^o) \tag{5}$$

and

$$T_{eff}^4 = \frac{F_\odot}{\sigma} = \frac{L_\odot}{4\pi R_\odot^2 \sigma} = \frac{S_\odot}{\tan(0.267^o)^2 \sigma}, \tag{6}$$

in which:

$L_\odot$ (solar luminosity)
$S_\odot$ (solar constant) = 1,367 kWm^{-2}
$D_\odot$ (solar distance) = 149,6 $\times$ 10^9 m
$R_\odot$ (solar radius) = 959,63 arcsec = $0{,}267^o$
σ = 5,67 $\times$ 10^{-8} $Wm^{-2}K^{-4}$

From this I could calculate T_{eff}, which is 5772 K. With this we can plot the solar spectrum (assuming a black body), in Mathematica

```
Plot[{Log[10, B[l, 5772]]}, {l, 2000*10^(-10), 10000*10^(-10)}]
```

to give the following figure:

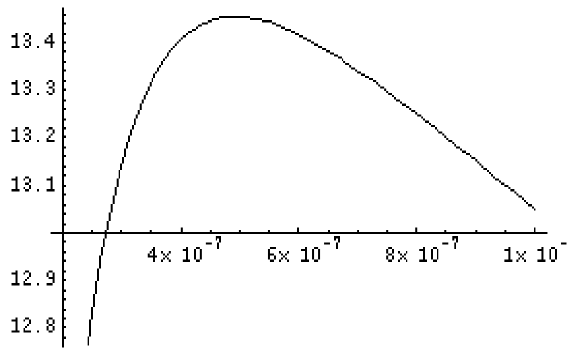

Figure 2: The Planck function for 5772 K: Flux Log(J/m^2 s) against the frequency ν (1/s).

From figure 2 I can find ν_{max} for the sun, which is about 5500 $\AA$.
Nevertheless this is a poor approximation of the solar spectrum because the sun

isn't a black body. In reality the spectrum would have a lot of absorption lines. The plot will give you an idea what the solar spectrum looks like, but not it's characteristics.

2 Solar temperatures

2.1 The temperature as a function of optical depth

In the second assignment we have to use the T_{eff} I have calculated to plot it against the optical depth. For this I used the formula:

$$T(\tau) = T_{eff}\left[\frac{3}{4}\left(\tau_o + \frac{2}{3}\right)\right]^{\frac{1}{4}}. \tag{7}$$

I have put this also in Mathematica:

```
K[t_]:=5772*((((t+2/3)*3)/4)^(1/4))
```

```
Plot[K[t],{t, 0,2}]
```

This resulted in figure 3.

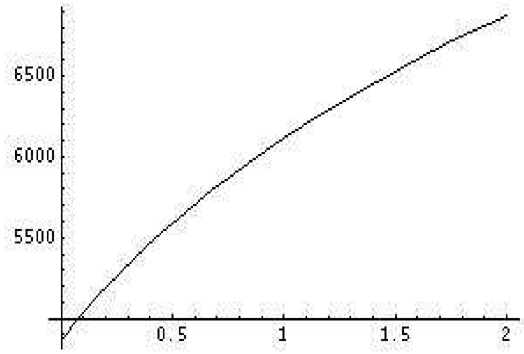

Figure 3: The temperature (K) against the optical depth τ.

2.2 The temperature at certain optical depths

The next part was the calculation the temperature at the surface (τ=0) and the temperature at $\tau=\frac{2}{3}$. I did this also in Mathematica, just filling in the formula, which resulted in T(τ=0) = 4854 K and T($\tau=\frac{2}{3}$) = 5772 K.

2.3 The Source Function as a function of frequency

According to the assumptions made, $S_\nu(\tau)=B_\nu(\tau)$. $B_\nu(T)$ is originally a function of T, but for T I have now a function

$$T = T_{eff}\left[\frac{3}{4}\left(\tau + \frac{2}{3}\right)\right]^{\frac{1}{4}}. \tag{8}$$

which can be used to fill in for T in $B_\nu(T)$, which results in the function

$$S(\nu,\tau) = \frac{2h\nu^3}{c^2}\left(e^{\frac{h\nu}{kT_{eff}\left(\frac{3}{4}\left(\tau+\frac{2}{3}\right)\right)^{\frac{1}{4}}}} - 1\right)^{-1} \tag{9}$$

with $T_{eff} = 5772$ K. With this function it is easy to plot it $S(\nu,\tau)$ against the frequency (Figure 4) in Mathematica:

```
S[v_, t_] := (2*h*v^3)/((Exp[(h*v)/(k*(5772*(3/4(t + 2/3))^(1/4)))] -
1)*c^2)

Plot[{Log[10, S[v, 0.001]], Log[10, S[v, 2/3]], Log[10, S[v, 10]]},
     {v, c/(2000*10^(-10)), c/(10000*10^(-10))}]
```

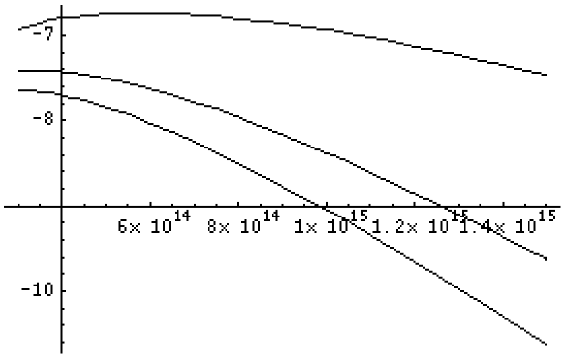

Figure 4: The Source function Log(J/s m^2) against the frequency ν (1/s) at the optical depths $\tau = 0.001$, 2/3 and 10.