

# Determining the Hubble constant with modified Newtonian dynamics

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# 1 Introduction

There are many ways to determine the value of the Hubble constant. One way is by dividing the systemic velocity of a galaxy by its distance. The relative distances to galaxies have not been known to within an accuracy, typically, of 25%, causing the value of the Hubble constant to be uncertain and having a large error as well.

In this Klein Onderzoek I use a new way of determining the distance to a galaxy by using the distance as a free parameter to make a best fit to the rotation curve of a galaxy. The fit is made with the use of Modified Newtonian Dynamics (MOND), an alternative theory explaining the asymptotic behaviour of observed rotation curves in its outer parts without having to resort to dark matter to solve the problem of its behaviour.

The MOND theory was proposed in three consecutive articles by Milgrom (1983). It basically adjusts the physics in the outer region of the rotation of a galaxy, where the law of gravity or inertia assumes a specific nonstandard form when the acceleration becomes extremely small when distances between stars are so large that the gravitational force is extremely small. This change occurs below a certain acceleration, named  $a_0$ , which is the one parameter of the theory and is the second free parameter I have used for my fits.

Two series of fits have been done. In the first series three free parameters were used: the distance,  $a_0$  and the mass of the disk, the common fitting parameter used for rotation curve fits. For the second series of fits, the different values for  $a_0$  of the first series were averaged and that average was made a constant to make a fit with only two parameters: the distance and the mass of the disk. The new disk masses I found have been converted to mass-to-light ratios in the results.

The reason for using  $a_0$  as a free parameter and as a constant afterwards is because previous determinations of the value of  $a_0$  (Begeman, Broeils & Sanders 1991, hereafter BBS; Bottema et. al 2002) were all based on a value of  $H_0$ , so it was necessary to remove this  $H_0$  dependency of  $a_0$  to get an independant distance fit. The exact value of  $a_0$  is still being determined. In general  $a_0$  is assumed to be a constant and the two fit series will shine a light on that issue as well, being able to compare the consequences on the fitting results in the different use of  $a_0$ . MOND itself is still being developed and tested. Although the research on it is small compared to the research on dark matter, it is one of the better alternatives to the rotation curve problem.

This research focusses on the rotation curves of disc galaxies which require very few parameters and have less possible irregularities in their structure, but with using a large dynamic range in size, mass, surface density and acceleration of the galaxies.

The main goal of this research is to see if making distance fits of galaxy rotation curves with MOND is a way to get more accurate distances of galaxies and as a consequence of that, more accurate values for the Hubble constant. Four differently measured systemic velocities were used to get a better idea of the spread of the  $H_0$  values. The idea behind determining the Hubble constant through MOND is to see whether it is consistent with the current cosmological theory or can maybe even contribute to the theory.

## 2 The sample

Twelve galaxies were used for this sample, their properties, used for the fit, are listed in Table 1. The sample data were provided by Sanders, those being the original data used in the reference articles by Sanders & Verheijen (1998, hereafter S&V) and Sanders (1996). Eight galaxies (nos. 1-8 in Table 1) are members of the Ursa Major Cluster (UMC), taken from S&V. Since these galaxies are closely related their distance is assumed to be 15.5 Mpc with a dispersion of 1–2 Mpc, following Tully & Verheijen (1997, hereafter T&V). S&V did a recalculation of the distance to 15.2 Mpc with a dispersion of 3 Mpc. They fixed the distance for their MOND fit at 15.5 Mpc, thus I am using that value for the comparison. Their dispersion is also large enough to fit in the

range of the T&V distance. The stellar contribution to the rotation curve was derived from the intensity of the galaxy in the near-infrared  $K'$ -band (around 22.200 Å), this because the scatter in M/L is very small and the mean M/L is about 1 in solar units, with a dispersion of 30%.

The four other galaxies (nos. 9-12 in Table 1) are taken from a sample with a wide range of galaxies of another MOND test (Sanders 1996). The stellar data from this sample are in the  $B$ -band (around 4400 Å), which is less reliable than redder bands as a tracer of the true radial distribution of the dominant stellar population, subject as it is to recent star formation and the effects of differential dust obscuration. The distance dispersion of these galaxies are not very clear. Galaxies 10 and 11 have distances based on the Hubble constant, which has its own uncertainty and is also an issue in this research. Galaxy 9's distance was taken from Sanders (1996), but references set the distance between 48 and 60 Mpc, which creates a large dispersion. The value of 48 was taken by using  $H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Knowing  $H_0$  to be smaller, this would yield a much larger distance. References to the distance of galaxy 12 set the adopted distance of 10.1 Mpc as the maximum distance, while it was still possible that the distance might even be only 3.2 Mpc. I set the dispersion at 5 Mpc, a value that I found referred to most often. These uncertainties would make most distances deviations in the MOND distance fit acceptable. Overall the distance dispersion for the sample galaxies are quite large, hence allowing deviations to the adopted distance to be possible while fitting.

The galaxy masses have been used so that the scaled masses of the fit wouldn't be too far off. The galaxies are generally of quite late morphological type (Sc to Sm), and the effects of a separate bulge component with a spheroidal shape and separate M/L are minimal, hence only the disk component is needed to make the fit. Galaxies 1-9 are mostly Low Surface Brightness (LSB) galaxies.

TABLE 1  
GALAXY PROPERTIES

no.	Galaxy	Type	$M_d$ ( $10^9 M_\odot$ )	$L$ ( $10^9 L_\odot$ )	$D$ (Mpc)	$\pm$ (Mpc)
1	NGC 4010	SBd	8.6	12.0	15.5	2.0
2	NGC 4183	Scd	5.9	7.3	15.5	2.0
3	UGC 6399	Sm	2.07	2.1	15.5	2.0
4	UGC 6446	Sd	1.17	1.4	15.5	2.0
5	UGC 6667	Scd	2.49	2.8	15.5	2.0
6	UGC 6917	SBd	5.41	4.2	15.5	2.0
7	UGC 6983	SBcd	5.65	3.4	15.5	2.0
8	UGC 7089	Sdm	0.92	2.1	15.5	2.0
9	UGC 128 <sup>c</sup>	Sdm	5.7	5.2	56.4	3.6
10	NGC 1003 <sup>c</sup>	Scd	3	15.0	11.8	2.0 <sup>a</sup>
11	NGC 5033 <sup>c</sup>	Sc	88	19.0	11.9	2.0 <sup>b</sup>
12	NGC 6946 <sup>c</sup>	SABcd	27	53.0	10.1	5.0

<sup>a</sup> Distance through the radial velocity with respect to the Virgo Cluster with  $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . The error is dependent of the accuracy in determining the systemic velocity and  $H_0$ , the latter being the goal of this research. No error was given for this distance, but taking an dispersion of 10 on  $H_0$ , which is reasonable, a dispersion can be calculated.

<sup>b</sup> No error was found on this distance either, although it was also calculated with the use of the same  $H_0$  as in <sup>a</sup>, but not which systemic velocity was used. With a distance similar to NGC 1003 the same error was used.

<sup>c</sup>  $B$ -band based data.

All necessary data was provided for: The observed rotation curve data, the intensity and the gas (neutral hydrogen) density of the galaxy over its radius. The observed curves were measured in the 21 cm line of neutral hydrogen, this because the rotation curves extend well beyond the optical image of the galaxy, where the discrepancy is large, and because gas on very nearly circular orbits is the most precise probe of the radial force law in the limit of low acceleration.

### 3 The procedure

To make the MOND fit rotation curves a package named GIPSY, developed here at the Kapteyn Institute, was used. GIPSY already contains the option to do a MOND fit when sufficient data are provided.

For the fit it is assumed that the azimuthally averaged radial profile of  $K'$ -band emission precisely traces the stellar mass distribution in an infinitesimally thin, axisymmetric disk. Assumed is thus that there is no variation of the stellar M/L within a given galaxy, and the entire galaxy mass, including any possible bulge component, is distributed in a thin disk. The gas mass distribution is assumed to lie in a thin disk and to be traced by the mean radial distribution of neutral hydrogen. The neutral hydrogen surface density is increased by a factor of 1.3 to account for primordial helium. This of course neglects any contribution of molecular gas, which, in the absence of more detailed information, is assumed to be distributed as the luminous component. Thus, given the distribution of observable matter, the Newtonian gravitational acceleration  $\mathbf{g}_N$  is calculated in the usual way (see Begeman 1987).

The modified dynamics post that the true gravitational acceleration  $\mathbf{g}$  is related to the Newtonian acceleration as

$$\mu(g/a_0)\mathbf{g} = \mathbf{g}_N , \quad (1)$$

where  $a_0$  is the MOND acceleration parameter and

$$\mu(x) = x(1+x^2)^{-1/2} . \quad (2)$$

This commonly assumed form as the appropriate asymptotic behavior, yielding Newtonian dynamics in the high-acceleration limit and MOND dynamics in the low-acceleration limit of the form  $g = (g_n a_0)^{1/2}$  (Milgrom 1983).

The predicted rotation curve is determined by setting the true gravitational acceleration equal to the centripetal acceleration, as  $r$  becomes large,

$$v^4 = GMa_0. \quad (3)$$

This is then fitted in a least square program (GIPSY) to the observed rotation curve by adjusting the mass of the disk, the one free parameter of the fitting algorithm. In this research the distance to the galaxy has also been made a free parameter, but this had to be done before any disk fitting could be done and which was done by fitting in steps, but with high accuracy. The acceleration parameter  $a_0$  was in the first series of fits also a free parameter and the second series fixed. The reasons for this have been explained in Section 1. Normally in GIPSY it is already set as a constant, although you can manually adjust the value.

## 4 Results

The results are shown in Table 2 and Tables 3 to 6, and in Figures 1–12. Two series of MOND fits were done. In the first series three free parameters were used: distance,  $a_0$  and the mass of the disk. In the second series, the twelve values for  $a_0$  were averaged. This average was then used as a fixed value for  $a_0$  in the fit, so that there were only two free parameters left: the distance and the mass of the disk.

In Table 2 each parameter has been given two columns. The first column shows the MOND fit with three parameters, the second column the MOND fit with two parameters. The lower the  $\chi^2$ , the better the fit. Although the value of  $\chi^2$  doesn't play a big role for the results, it has been added to show the effects of using  $a_0$  as a fixed and a free parameter and also to show how well MOND is able to make fits to observed rotation curves.

Fitting the galaxies with distance as a free parameter was done by taking steps of 0.1 Mpc per fit, which will really be very close to the best fit distance and small enough not to make much difference when calculating the Hubble constant. Scaling with  $a_0$  was done in steps of 50, again small enough to be very close to the best value, although the sensitivity of accurately fitting  $a_0$  varies per sample. This sensitivity can be neglected because finding the true value of  $a_0$  is not the goal of this research and its influence would not hamper with the final results of this research. There was a similar but less prominent sensitivity of the accuracy in determining the distance in the three parameter fit. These varying sensitivities make it hard to give error margins, although those remain reasonably small, considering that the best fits could be determined with much certainty. The  $a_0$  as it is used in GIPSY works with different units,  $\text{km s}^{-1} \text{s}^{-1} \text{kpc}^{-1}$ , which are equal to the standard units when converted.

TABLE 2  
3 & 2 PARAMETER MOND-FIT RESULTS

Galaxy	$D^a$ (3p)	$D^a$ (2p)	$M_d/L^b$ (3p)	$M_d/L^b$ (2p)	$\chi^2$ (3p)	$\chi^2$ (2p)	$a_0^c$ (3p)	$\bar{a}_0^d$ (2p)
NGC 4010	17.7	17.7	0.193	0.524	13.453	17.633	9384	4055
NGC 4183	12.9	12.1	1.979	1.318	2.685	13.427	2134	4055
UGC 6399	15.2	15.2	0.846	0.951	1.295	1.315	4534	4055
UGC 6446	11.3	11.2	3.138	1.776	4.205	10.903	2334	4055
UGC 6667	15.5	16.3	0.415	0.709	4.078	5.295	7034	4055
UGC 6917	15.7	15.7	1.120	1.151	8.142	8.162	4184	4055
UGC 6983	16.0	16.0	2.774	1.442	18.024	30.686	1734	4055
UGC 7089	15.4	15.5	0.264	0.385	1.167	1.377	5134	4055
UGC 128 <sup>e</sup>	55.9	55.4	1.312	1.019	5.567	5.980	3384	4055
NGC 1003 <sup>e</sup>	11.8	11.0	0.220	0.199	131.309	134.028	3284	4055
NGC 5033 <sup>e</sup>	11.6	11.5	5.323	4.564	151.356	200.226	2934	4055
NGC 6946 <sup>e</sup>	9.1	9.0	0.889	0.606	85.821	179.705	2484	4055

<sup>a</sup>  $D$  in Mpc

<sup>b</sup>  $M_d/L$  in  $M_\odot/L_\odot$ .

<sup>c</sup>  $a_0$  in  $\text{km s}^{-1} \text{s}^{-1} \text{kpc}^{-1}$ , which is equal to  $(a_0/3734) \times 1.2 \times 10^{-8} \text{ cm s}^{-2}$ .

<sup>d</sup>  $\bar{a}_0$  = average  $a_0$  over the 12 sample galaxies, equal to  $1.3 \times 10^{-8} \text{ cm s}^{-2}$ .

<sup>e</sup>  $B$ -band based results.

3p = 3 parameter fit result, 2p = 2 parameter fit result.

For some galaxies the measured distance hardly changed when changing to the two parameter fit. For other galaxies the distance changed a lot, more than 0.5 Mpc. Searching for the best fit resulted in a wider range of values for the mass of the disk component and  $a_0$ . Nevertheless most values remained within an acceptable range. Obviously, using the average  $a_0$  gave less to not very good fits, but in most cases the fits remained still decent.

## 4.1 Rotation curves

The plots of the rotation curves with the MOND fits have been set per galaxy with the two series next to each other, with additional comments to the results of the fits.

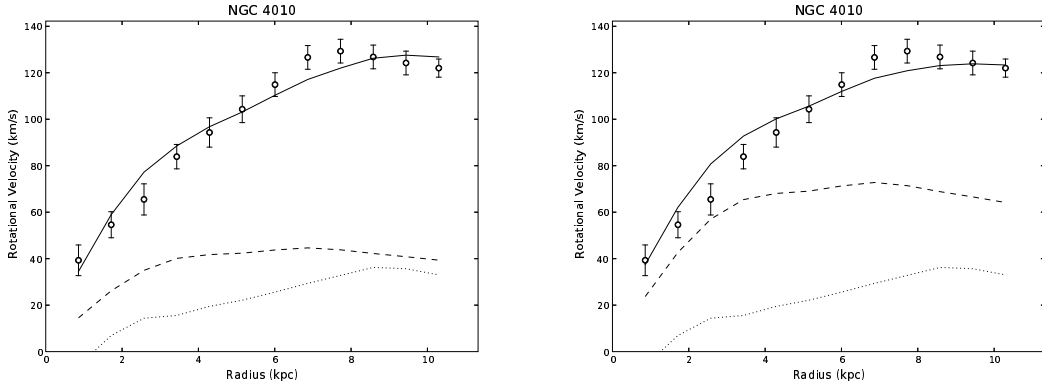


Figure 1: MOND fits on observed rotation curves. The dotted line is the HI contribution, the dashed line the stellar contribution and the solid line the MOND fit. Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $3.01 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value at  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**NGC 4010** This galaxy has a rotation speed that keeps rising and suddenly breaks off at the outer part. MOND predicted a smoother transition, which was not the case. The  $\chi^2$  is reasonable and the same counts for the fit. The distance is with 17.7 much larger and just outside the T&V range. It is in the range of the wider S&V range. The value of  $a_0$  in the three parameter fit is almost three times as high as literature values, making it very extreme. The stellar M/L is lowest of all galaxies. The two parameter fit did not change much. The distance remained the same, although  $a_0$  was less than half of the best fit value.

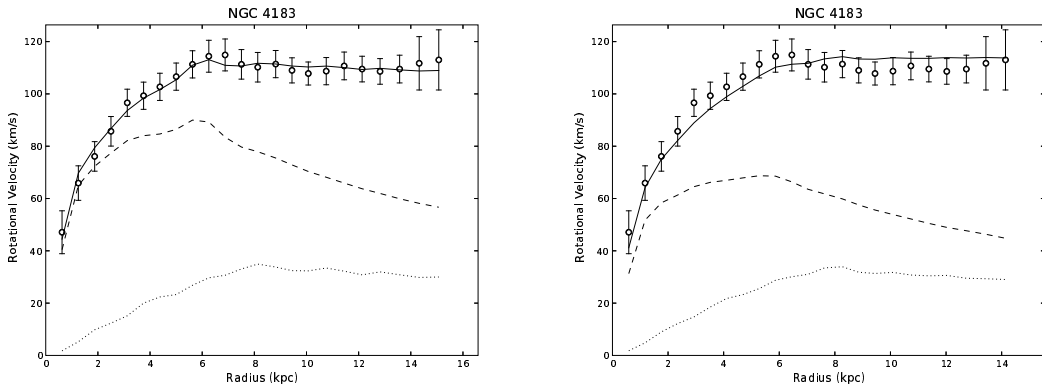


Figure 2: For the explanation of the lines, see Fig. 1. Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $0.68 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value of  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**NGC 4183** The best distance fit appeared outside the range for the dispersion for UMC galaxies, and at a much shorter distance. The two parameter fit resulted in an even shorter distance with a lesser but still decent fit.

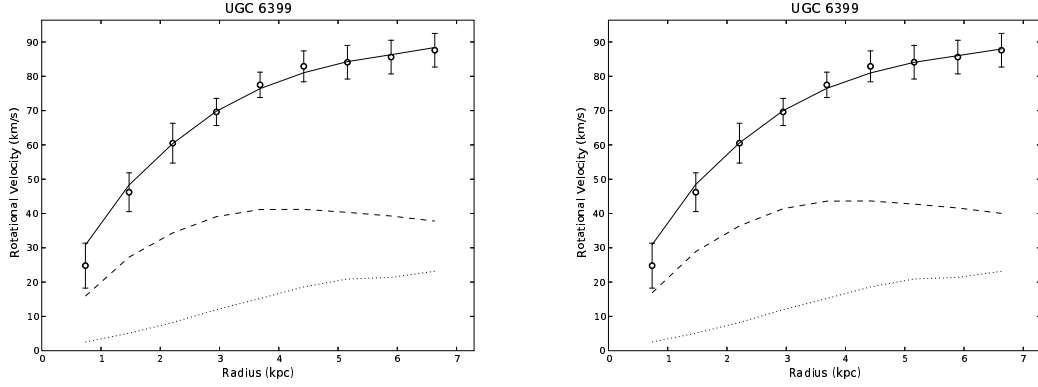


Figure 3: For the explanation of the lines, see Fig. 1. Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $1.45 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value of  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**UGC 6399** MOND fits this curve really well, with a very low  $\chi^2$  and a distance just under the adopted average. The two parameter fit did not change much.

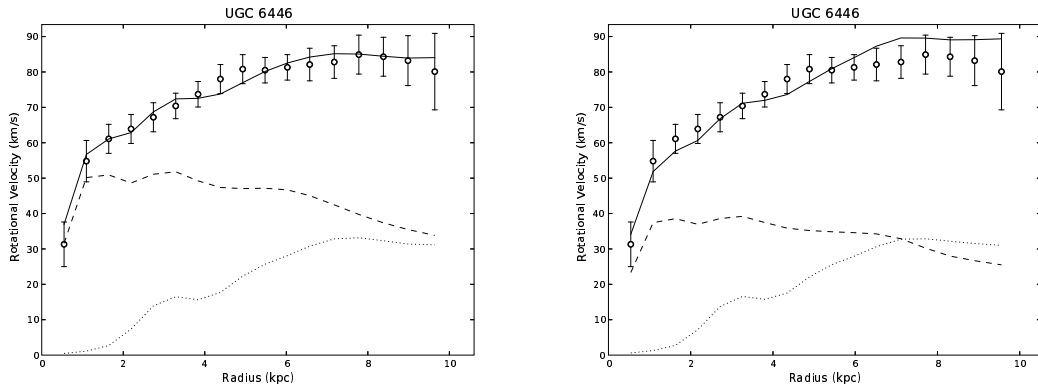


Figure 4: For the explanation of the lines, see Fig. 1. Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $0.73 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value of  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**UGC 6446** The distance to this galaxy is out of the dispersion range set for the UMC. In S&V this fact has already been noted, although they set the best distance fit at 8-9 Mpc, much lower than the distance I found. The systemic velocity of UGC 6446 is much lower than the UMC mean of  $950 \text{ km s}^{-1}$  (taken from T&V, with a dispersion of  $150 \text{ km s}^{-1}$ ), hence this shorter distance was expected. The stellar M/L is quite high. The two parameter fit is less good, but the distance remains about the same, with the stellar M/L is much lower.



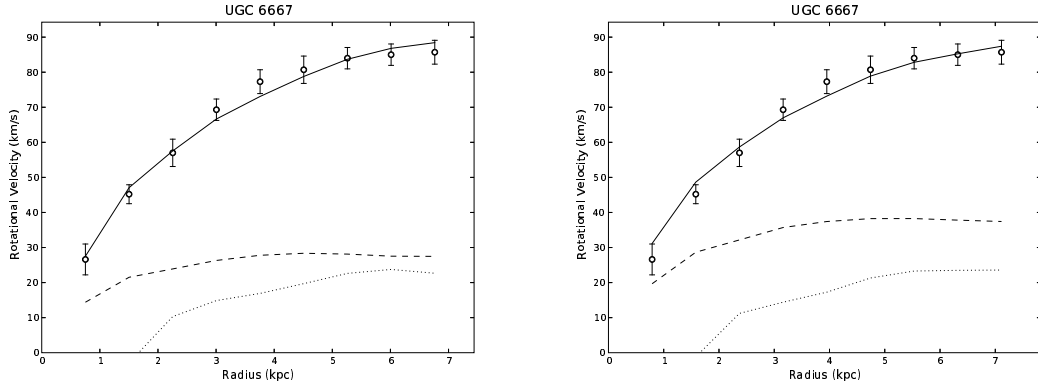


Figure 5: For the explanation of the lines, see Fig. 1. Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $2.26 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value of  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**UGC 6667** The distance is at the average distance of the UMC, but in the two parameter fit it becomes much larger, but still within range and a good fit in both cases.

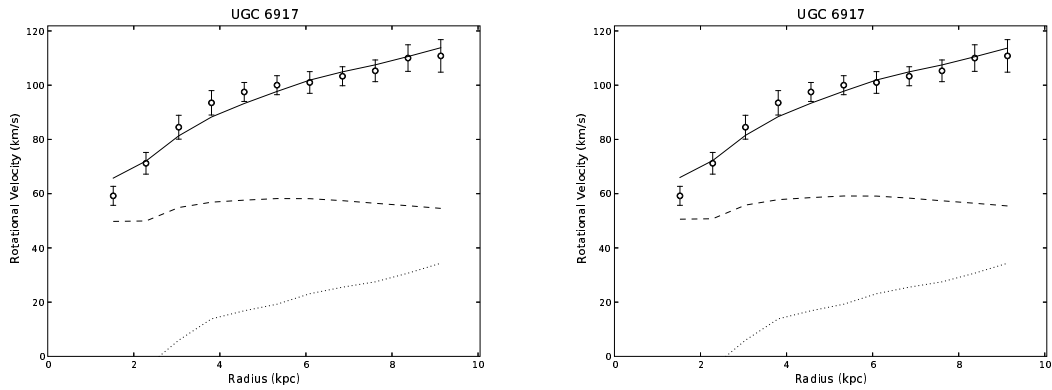


Figure 6: For the explanation of the lines, see Fig. 1. Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $1.34 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value of  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**UGC 6917** A good fit overall, with hardly any differences between the three and the two parameter fits.

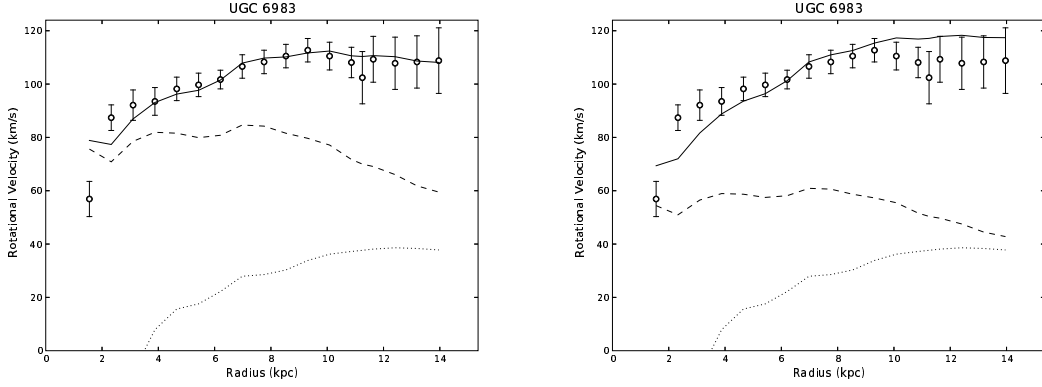


Figure 7: For the explanation of the lines, see Fig. 1. Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $0.55 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value of  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**UGC 6983** The best distance is higher than the mean for the UMC. The two parameter fit gives a low  $a_0$  and a high stellar M/L. The two parameter fit is much less good, but the distance remained the same. The stellar M/L is much lower.

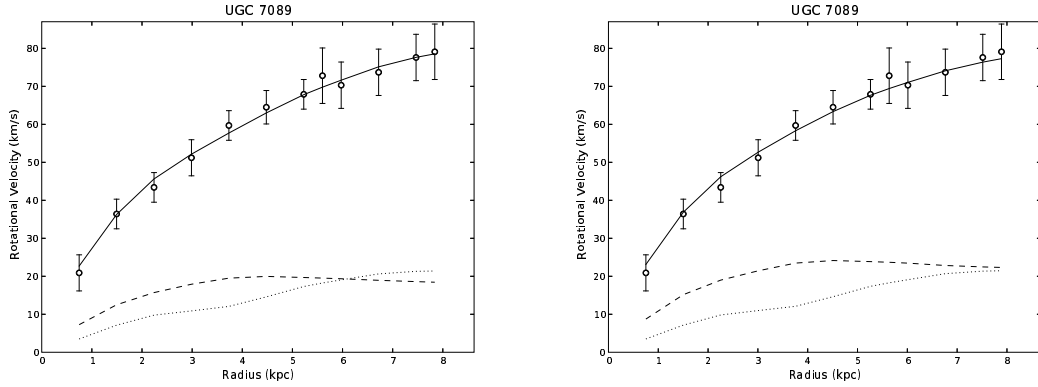


Figure 8: For the explanation of the lines, see Fig. 1. Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $1.64 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value of  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**UGC 7089** The distance is again close to the mean and the fit is very good. The two parameter fit does not change much.

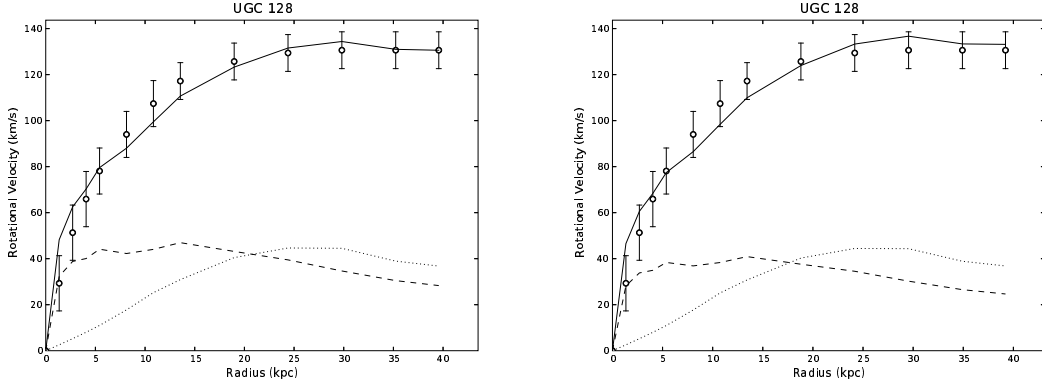


Figure 9: For the explanation of the lines, see Fig. 1. Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $1.09 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value of  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**UGC 128** The distance for the fit was quite shorter, but with the large dispersion still within range. The two parameter fit did not change much, but the distance was twice as much shorter than the three parameter fit.

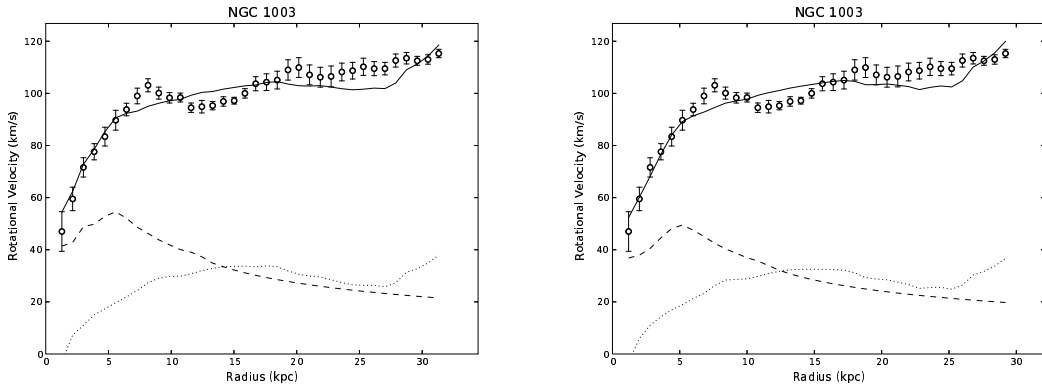


Figure 10: For the explanation of the lines, see Fig. 1. Left: Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $1.06 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value of  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**NGC 1003** Not a very good fit, but this is probably caused by the influence of the spiral arms on the velocity speed data. The three parameter fit gave the same distance as literature values, but the two parameter fit set the distance much shorter, while not effecting the other values much.

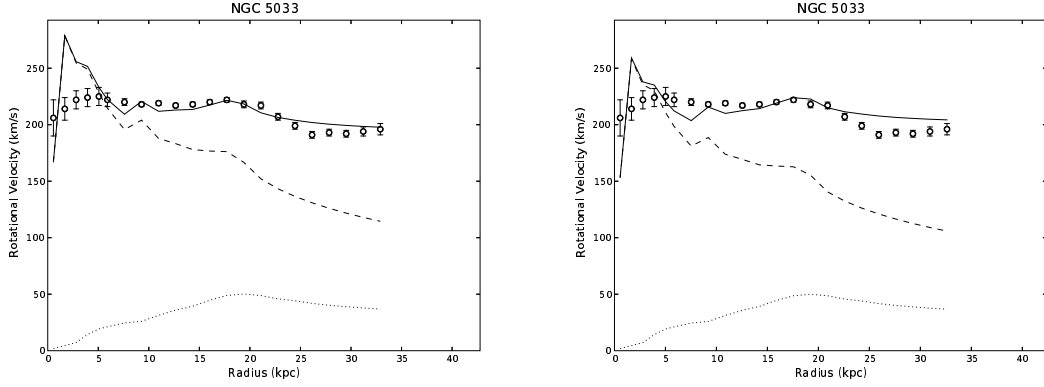


Figure 11: For the explanation of the lines, see Fig. 1. Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $0.95 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value of  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**NGC 5033** A peculiar peak in the early part of the fit mainly ruined the good fit of this galaxy. The peak is solely caused by the stellar contribution, which seems to be smaller in the fit by Sanders (1996). MOND mainly provides an alternative behaviour in the outer parts and there the fit is much better. The existence of a bulge, which is not taken into account for this fit, may be the cause of the peak. The distance is a bit shorter, but still within range. The two parameter fit does not change much, but the peak is much smaller. Nevertheless the fit is much worse. There is a very high stellar M/L for both fits.

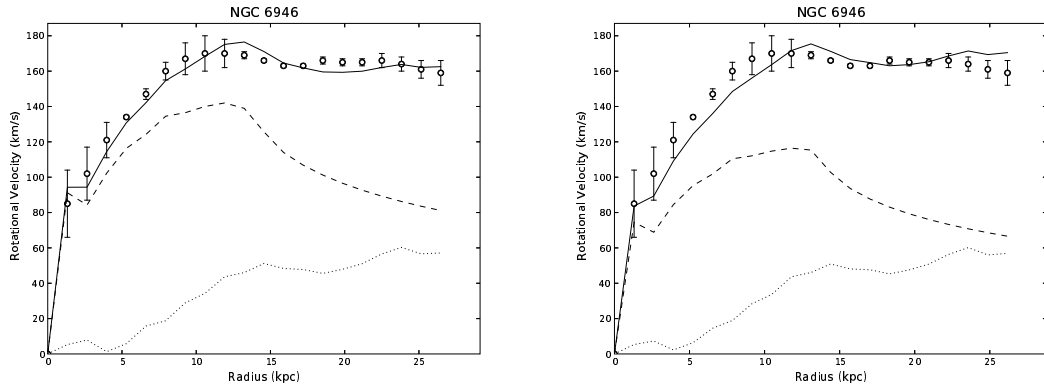


Figure 12: For the explanation of the lines, see Fig. 1. Left: MOND fit with  $a_0$  used as a free parameter, giving a value of  $0.80 \times 10^{-8} \text{ cm s}^{-2}$ . Right: MOND fit with  $a_0$  fixed at the average value of  $1.30 \times 10^{-8} \text{ cm s}^{-2}$ .

**NGC 6946** A reasonable fit, giving a much shorter distance. Knowing that the 10.1 Mpc used is considered a maximum value with a minimum value of 5 Mpc, this result is alright. The two parameter fit made the fit much worse, although the plot doesn't show it that clearly, but it did not change the distance much.

## 4.2 The Hubble constant

Table 3 shows the Hubble constant calculated on the MOND distances of the three parameter fits and four systemic velocities that are used to calculate  $H_0$ . The data for the four velocities were taken from the HyperLeda<sup>1</sup> internet database.  $H_0$  is calculated with the simple and common relationship  $H_0 = v/D$ , with  $v$  the systemic velocity of an object and  $D$  the distance to the object from Earth. Table 4 shows the average  $H_0$ 's over the sample galaxies for the three parameter fit, using the results of Table 3.

The current best value for  $H_0$  is taken from WMAP (Wilkinson Microwave Anisotropy Probe), which set it a  $71 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

TABLE 3  
THE HUBBLE CONSTANT - 3 PARAMETER FIT

Galaxy	3p- $D^a$	$v_{LG}^b$	$H_0^c$	$v_{GSR}^d$	$H_0^c$	$v_{vir}^e$	$H_0^c$	$v_{CMBR}^f$	$H_0^c$
NGC 4010	17.7	958	54.12	958	54.12	1125	63.56	1117	63.11
NGC 4183	12.9	968	75.03	976	75.66	1143	88.60	1157	89.69
UGC 6399	15.2	867	57.04	854	56.18	1021	67.17	996	65.53
UGC 6446	11.3	728	64.42	710	62.83	876	77.52	826	73.09
UGC 6667	15.5	1051	67.81	1040	67.10	1207	77.87	1170	75.48
UGC 6917	15.7	980	62.42	974	62.04	1140	72.61	1108	70.57
UGC 6983	16.0	1159	72.44	1148	71.75	1314	82.13	1261	78.81
UGC 7089	15.4	807	52.40	815	52.92	983	63.83	1004	65.19
UGC 128 <sup>g</sup>	55.9	4820	86.23	4716	84.36	4680	83.72	4230	75.67
NGC 1003 <sup>g</sup>	11.8	864	73.22	747	63.31	759	64.32	429	36.35
NGC 5033 <sup>g</sup>	11.6	896	77.24	926	79.83	1082	93.28	1107	95.43
NGC 6946 <sup>g</sup>	9.1	336	36.92	274	30.11	320	35.16	-119	-13.08

<sup>a</sup> in Mpc

<sup>b</sup> in  $\text{km s}^{-1}$ . Radial velocity with respect to the Local Group.

<sup>c</sup> in  $\text{km s}^{-1} \text{ Mpc}^{-1}$ .

<sup>d</sup> in  $\text{km s}^{-1}$ . Radial velocity with respect to the Galactic Standard of Rest.

<sup>e</sup> in  $\text{km s}^{-1}$ . Radial velocity with respect to the Virgo Cluster.

<sup>f</sup> in  $\text{km s}^{-1}$ . Radial velocity with respect to the Cosmic Microwave Background Radiation.

<sup>g</sup> B-band based results.

The error in the best distance fit would only have a small influence in the error of the result of the  $H_0$  values. Since I have no errors for the systemic velocities, a decent error for the resulting  $H_0$  was not possible, so I left that out. Considering the wide range of  $H_0$  values I obtained, which created large individual errors, the errors for the velocities would, because of that, be rendered so much smaller that their effect on the final error for the average  $H_0$ 's would be neglectable small.

TABLE 4  
THE AVERAGE HUBBLE CONSTANT - 3 PARAMETER FIT

A: Total sample average $H_0$			B: UMC sample average $H_0$		
$\overline{H}_{0LG}$	64.94	$\pm 10.39$	$\overline{H}_{0LG}$	63.21	$\pm 6.72$
$\overline{H}_{0GSR}$	63.35	$\pm 10.32$	$\overline{H}_{0GSR}$	62.83	$\pm 6.51$
$\overline{H}_{0vir}$	72.48	$\pm 11.39$	$\overline{H}_{0vir}$	74.16	$\pm 7.37$
$\overline{H}_{0CMBR}$	64.65	$\pm 17.93$	$\overline{H}_{0CMBR}$	72.68	$\pm 6.58$

All  $H_0$ 's in  $\text{km s}^{-1} \text{ Mpc}^{-1}$

<sup>1</sup><http://leda.univ-lyon1.fr/>

Because 8 galaxies are from one cluster, an extra set of averages (see Tables 4B and 6B) was calculated leaving out the four other ones which had more strange numbers to provide a reference for earlier calculated values, although those were from a more complete UMC sample

Table 5 shows the Hubble constant calculated on the MOND distances of the two parameter fits. The systemic velocities are the same as used in Table 3.

TABLE 5  
THE HUBBLE CONSTANT - 2 PARAMETER FIT

Galaxy	2p- $D^a$	$v_{LG}^b$	$H_0^c$	$v_{GSR}^d$	$H_0^c$	$v_{vir}^e$	$H_0^c$	$v_{CMBR}^f$	$H_0^c$
NGC 4010	17.7	958	54.12	958	54.12	1125	63.56	1117	63.11
NGC 4183	12.1	968	80.00	976	80.66	1143	94.46	1157	95.62
UGC 6399	15.2	867	57.04	854	56.18	1021	67.17	996	65.53
UGC 6446	11.2	728	64.71	710	63.39	876	78.21	826	73.75
UGC 6667	16.3	1051	64.48	1040	63.80	1207	74.05	1170	71.78
UGC 6917	15.7	980	62.42	974	62.04	1140	72.61	1108	70.57
UGC 6983	16.0	1159	72.44	1148	71.75	1314	82.13	1261	78.81
UGC 7089	15.5	807	52.06	815	52.58	983	63.42	1004	64.77
UGC 128 <sup>g</sup>	55.4	4820	87.00	4716	85.13	4680	84.48	4230	76.35
NGC 1003 <sup>g</sup>	11.0	864	78.55	747	67.91	759	69.00	429	39.00
NGC 5033 <sup>g</sup>	11.5	896	77.91	926	80.52	1082	94.09	1107	96.26
NGC 6946 <sup>g</sup>	9.0	336	37.33	274	30.44	320	35.56	-119	-13.22

<sup>a</sup> in Mpc

<sup>b</sup> in km s<sup>-1</sup>. Radial velocity with respect to the Local Group.

<sup>c</sup> in km s<sup>-1</sup> Mpc<sup>-1</sup>.

<sup>d</sup> in km s<sup>-1</sup>. Radial velocity with respect to the Galactic Standard of Rest.

<sup>e</sup> in km s<sup>-1</sup>. Radial velocity with respect to the Virgo Cluster.

<sup>f</sup> in km s<sup>-1</sup>. Radial velocity with respect to the Cosmic Microwave Background Radiation.

<sup>g</sup> B-band based results.

Table 6 shows the average  $H_0$ 's for the two parameter fit, using the results from Table 5. The comments are the same as those of Table 4.

TABLE 6  
THE AVERAGE HUBBLE CONSTANT - 2 PARAMETER FIT

A: Total sample average $H_0$			B: UMC sample average $H_0$		
$\overline{H}_{0_{LG}}$	65.67	$\pm 11.26$	$\overline{H}_{0_{LG}}$	63.41	$\pm 7.00$
$\overline{H}_{0_{GSR}}$	64.04	$\pm 10.96$	$\overline{H}_{0_{GSR}}$	63.07	$\pm 6.84$
$\overline{H}_{0_{vir}}$	73.23	$\pm 11.32$	$\overline{H}_{0_{vir}}$	74.45	$\pm 7.86$
$\overline{H}_{0_{CMBR}}$	65.19	$\pm 17.90$	$\overline{H}_{0_{CMBR}}$	72.99	$\pm 7.30$

All  $H_0$ 's in km s<sup>-1</sup> Mpc<sup>-1</sup>

## 5 Discussion

Three free parameters were used during the MOND fitting. I will treat them in order of fitting. In the section 5.4 the results for Hubble constant will be discussed.

### 5.1 The best fit distances

Fitting MOND on the distance provided in two-thirds of the sample very different distances compared to the literature distance, although most were in the dispersion range, except for two galaxies from the UMC sample, which showed large discrepancies.

Doing a new distance fit with two parameters based on the average of  $a_0$ , one of the free parameter of the three parameter fits, gave unpredictable results. In eight cases the fitted distance changed hardly or not at all. In four cases the distances changed considerably in the range of 0.5 to 0.8 Mpc.

In the case of the 8 UMC galaxies, 5 MOND distances were within the different literature distance ranges. The shorter distance of UGC 6446 was expected, although not as short as S&V had predicted. The large deviations of NGC 4010 and NGC 4183 (especially the two parameter fit), in different directions, are odd, considering that their systemic velocities are close together in all four cases. They are out of the range of the T&V dispersion, but just within range of the wider S&V range.

The shorter distance for UGC 128 would not have been expected when looking at the systemic velocities, since this would raise the Hubble constant for this galaxy. The distance of NGC 1003 stayed at the literature distance in the three parameter fit, but became much shorter in the two parameter fit. The NGC 5033 distances are a bit shorter than the literature distance, but still within range.

The adopted distance of NGC 6946 was the most odd one of the sample, being a maximum distance. Low systemic velocities hinted already that the distance should be much smaller, so the result of 9.0 Mpc is very acceptable.

### 5.2 The acceleration parameter $a_0$

Milgrom, the developer of MOND has argued to keep the acceleration parameter  $a_0$  at a fixed value, seeing it as a possible fundamental constant. The value for  $a_0$  was set in the order of  $1 \times 10^{-8} \text{ cm s}^{-2}$ . Bottema et al. (2002) have argued for a value of 0.9, while previously it was set at  $1.21 \pm 0.27$  (BBS).

The issues concerning  $a_0$  are still large and for this research two series of fits were done because of that. In the first series  $a_0$  was used as a free parameter, in the second series  $a_0$  was fixed at the average value of the  $a_0$ 's from the first series.

Using  $a_0$  as a free parameter gave a wide range of values, between 0.55 to 3.01. Two galaxies (NGC 4010 and UGC 6667) gave extreme values; about two and three times the normal value, while the other values remained reasonably close to literature values.

The average value thus became  $(1.30 \pm 0.23) \times 10^{-8} \text{ cm s}^{-2}$ : just above the BBS value and within its range. However, if the two extremes had not been included, the average value would have dropped to about  $1.03 \times 10^{-8} \text{ cm s}^{-2}$ ; a value much more reasonable to the recent research on the value of  $a_0$ . The question of doing so can be judged from looking at the change of the  $\chi^2$  of the two parameter fit. For NGC 4010 it is increased with 4 and for UGC 6667 1.5. In the case of UGC 6667 the influence on the fit is small and neglectable. For NGC 4010 the influence is more considerable, but still not large. From this can be concluded that these two extreme cases had a large influence on the value of  $a_0$  while the higher  $a_0$  this resulted in had not much influence on the goodness of their fit. Hence I can say the used average does not conflict with the claim for a (much) lower value of  $a_0$ .

TABLE 7  
THE FREE PARAMETER  $a_0$

Galaxy	$a_0^a$ ( $10^{-8}$ cm s $^{-2}$ )	Type
NGC 4010	3.01	SBd
NGC 4183	0.75	Scd
UGC 6399	1.45	Sm
UGC 6446	0.73	Sd
UGC 6667	2.26	Scd
UGC 6917	1.34	SBd
UGC 6983	0.55	SBcd
UGC 7089	1.64	Sdm
UGC 128 <sup>b</sup>	1.09	Sdm
NGC 1003 <sup>b</sup>	1.06	Scd
NGC 5033 <sup>b</sup>	0.95	Sc
NGC 6946 <sup>b</sup>	0.80	SABcd

<sup>a</sup>  $a_0$  scaled to the best fit  $\chi^2$ .

<sup>b</sup>  $B$ -band based results.

I looked for some relations of the scaled  $a_0$  and discovered some grouping. I made a plot ordering the galaxies according to the De Vaucouleurs scheme, which was also used in the S&V and Sanders (1996), ordering them from Sc to Sm galaxies, and setting  $a_0$  out onto them. The setting of SB and S was arbitrary and not of influence to the plot. The results were put in Figure 13.

Ignoring the two most deviating  $a_0$ 's, there is some notable grouping visible at the Sc–Scd range, giving lower  $a_0$  values. On the other side there is some grouping of Sd–Sm galaxies giving a bit higher  $a_0$ 's, although it can be said that the difference is small.

With only so few samples it's hard to conclude something from this plot. The question to be researched is if more open and irregular galaxies tend to have more higher  $a_0$ 's when using  $a_0$  as a free parameter, and if more closed galaxies tend to have lower values. The use of an extended sample of galaxies could tell if this grouping is just a coincidence or for real or that the behaviour of MOND physics is slightly different for different morphological types.

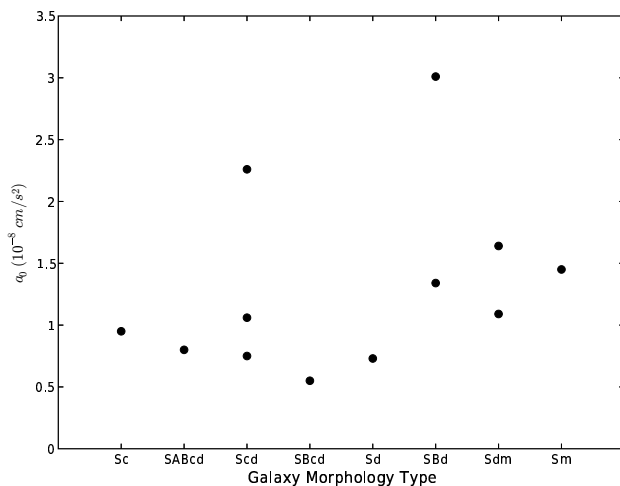


Figure 13:  $a_0$  over the range of galaxy morphological types, going from more closed spiral arms in the left to open/irregular on the right side.



### 5.3 The Mass-to-Light ratio

The sample consisted of only disk galaxies of which the bulge contribution to the total stellar contribution was neglectable and hence not needed as an extra fitting parameter. In the MOND fit there is no contribution of dark matter or a dark halo to make a good fit to the observed rotation curve. Hence the contribution of the stellar mass to the rotation curve is about equal to the total luminosity of the galaxy and thus the stellar mass over light ratio should be around 1 on average. The assumptions underlying the procedure are supported by this constancy of  $M/L$  and the absence of a significant contribution to the surface density by molecular gas that is distributed differently from the stars.

For the three and two parameter MOND fits the resulting mass-to-light ratio distributions for the sample galaxies have been plotted in Figure 14. The distribution is very wide, in the range  $0.220 - 5.323$ , although most ratios are below 2. Explanations for low values can be contributed to the possibility of starburst galaxies, while high values could be caused by cold gas and other components not taken into account for the fit. The left plot shows the results of the three parameter fit. The average  $M_d/L$  in the first series is 1.541, which is a bit high when you expect a value close to 1. The second fit series, with two free parameters, are spread much less and are on average getting closer to 1. The average of 1.220 is also better. Nevertheless the wide range of ratios are reasonable and consistent with population synthesis models. Only NGC 5033 remained far off from the ratios of the other galaxies. The possible existence of a bulge in this galaxy, creating an odd peak in the fit, could be the cause of that.

The closer grouping near 1 of the two parameter fit supports the claim for a constant  $a_0$ , although the sample is a bit small and the spread still large to be able to make a solid statement.

Comparing the results of the two parameter fit (for the three parameter fit most ratios are off a lot because of using  $a_0$  as a free parameter) with the reference articles gives very different ratios for NGC 4183 and UGC 6446, both with a different distance of respectively 3.3 and 4.3 Mpc. For NGC 4010, with 2.2 Mpc difference, the difference in ratio is more alike the other galaxies. More remarkable is that the four galaxies with data taken in the  $B$ -band stay close to the ratios of the reference articles even though three of the four have a distance that has changed with about 1 Mpc. The distance change does have its effect on the ratios, but from the sample there is no clear correlation recognizable, while you would expect it somewhat, because the fitted masses for the two parameter fit stayed close to the adopted masses (taken from the reference articles) that were used for the fit. So the change in distance (and hence the change in luminosity) would be the main parameter for the change in the mass-to-light ratio, but this does not clearly happen.

The new mass-to-light ratios are not much better or worse than those from the reference articles.

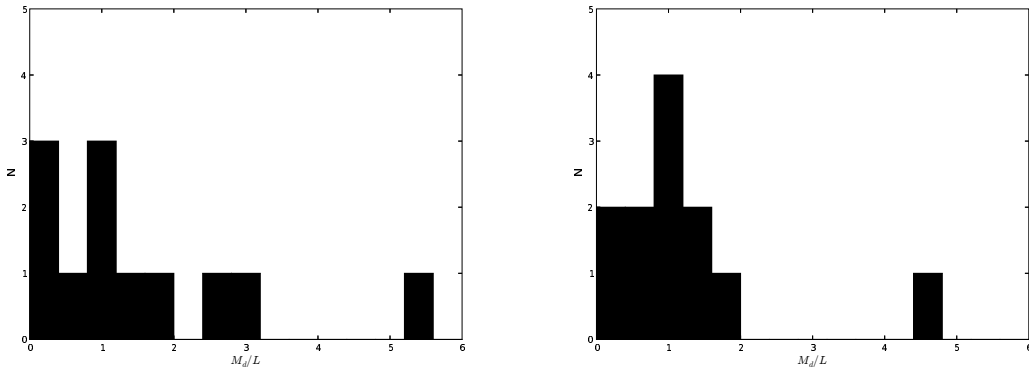


Figure 14: Left: The distribution of the  $M_d/L$  ratio for the three parameter fit with  $a_0$  as a free parameter. Right: The distribution of the  $M_d/L$  ratio for the two-parameter fit with a fixed  $a_0$ .

## 5.4 The Hubble constant

Tables 4 (three parameter fit) and 6 (two parameter fit) show the average values for the Hubble constant based on the four different systemic velocities used for the calculation. Tables 4A and 6A show the averages over the whole sample.

For the galaxies NGC 4183, UGC 128 and NGC 5033 some of their  $H_0$ 's are on the high side, but still within maximum boundaries. Oddly enough the CMBR provides much lower velocities for NGC 1003 and NGC 6946, which makes the  $H_0$  extremely low. This is caused by a systematic error for the CMBR velocities on short distances. Strangely enough this effect is not seen for NGC 5033 and instead gives a higher  $H_0$ , although  $v_{vir}$  is also large. The shorter distance of UGC 128 has a negative effect on the  $H_0$  values. NGC 6946 seems to have really low systemic velocities, explaining why there was a maximum adopted distance set for it and the large dispersion it had attributed to its distance. Even with the shorter MOND distance, the value of  $H_0$  remains far below acceptable values.

Overall the different values for  $H_0$  are in an acceptable range. The averages lie remarkably close together when considering that the systemic velocities did not show patterns when comparing the change of the different systemic velocities for each galaxy with each other. The dispersion is large and all cover each other, giving a range for  $H_0$  between 60 and 80  $\text{km s}^{-1} \text{Mpc}^{-1}$ , which is not bad, but does not provide much substantial conclusions, although most galaxies gave quite different distances compared to the literature values. It can mainly be said that the new distances fit their systemic velocities better, but not in all cases.

Comparing the four averages per systemic velocity shows that they are mostly close together, except for the  $H_{0_{vir}}$ 's, which is much higher and also closer to the WMAP value.

Based on their adopted distance and mean systemic velocity for the UMC, T&V calculated a  $H_0$  of 85  $\text{km s}^{-1} \text{Mpc}^{-1}$ . The systemic velocities of the individual galaxies vary a lot. Fitting on the distance created a chance to see if the MOND distance would correct differences in the  $H_0$  value. This is however not the case, although the expected distance correction for NGC 6446 provided a better  $H_0$  value. But these eight galaxies are only a very small sample of the UMC and can not make conclusions for the whole cluster. However, most  $H_0$ 's are below the T&V value.

Tables 4B and 6B show the average  $H_0$ 's if only the 8 UMC galaxies are taken into the calculation. This second set of averages was calculated because the UMC galaxies are closely related and the other four galaxies gave quite extreme results. The dispersions of the UMC averages are smaller and mainly the  $H_{0_{CMBR}}$  is much higher, coming closer to the WMAP value of 71.

All average  $H_0$ 's are within range of the WMAP value, although only the  $H_{0_{vir}}$  and  $H_{0_{CMBR}}$  are in the WMAP dispersion range.

## 6 Conclusions

MOND gives very good rotation curve fits in most cases when you look at the results. The sample consists of galaxies with large dispersions for their distances, which creates a necessity for a more accurate distance determination method. The galaxies of the UMC sample have not had a individual distance determination before; only an average was available.

In the MOND fits however, two distances were just out of the range set by literature values. There is no answer to the discrepancy of the two galaxies, both seeming to be average before the fitting. Except those two all found distances are acceptable new distance values.

The MOND distances provide in most cases reasonable values for  $H_0$ , those that are out of bounds already have awkward systemic velocities, even with the adopted distance, so the error could lie there. The use of MOND to measure more accurate distances is viable, but it can not be concluded yet that it is a better way to measure distances.

The stellar mass-to-light ratios for the fits are all consistent with popular synthesis models and supporting the MOND fit results. Using  $a_0$  as a free parameter created a wider range of ratio values, while using a constant  $a_0$  grouped the ratios more closer together near 1. Although both total average mass-to-light ratios are above 1, they are near enough to support the initial assumptions which say that there is no dark matter influencing the shape of the rotation curve and that because of that the sample galaxies should have a mass-to-light ratio that is near to 1.

Using  $a_0$  as a free parameter allows you to make a very good fit, but the effect of using a constant  $a_0$  on the fit (which will obviously makes a less good fit) is reasonably small. Seeing that and looking at the way a constant  $a_0$  gives stellar mass-to-light ratios closer to 1, it can be concluded that the claim for a constant  $a_0$  is justified, although the used sample is somewhat small. The average  $a_0$  value that was used as a constant in the two parameter fit was above literature values. However, this higher value was mainly caused by two extremes, and thus not undermining the claim of a value for  $a_0$  between 0.9 and  $1.2 \times 10^{-8} \text{ cm s}^{-1}$ .

The values for the Hubble constant that MOND generates through best distance fits are not much different from common results. Although somewhat below recent values for  $H_0$  they are still within range, setting the Hubble constant between 60 and 80  $\text{km s}^{-1} \text{ Mpc}^{-1}$ , thus remaining within common boundaries and consistent with recent results on measuring  $H_0$ . The sample contained some galaxies with odd data, thus giving odder results, but their influence was not large in the final results.

## Acknowledgements

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# APPENDIX

## A Python programs

Although two scripts were written in python, one of the two was a special script that was used in the GIPSY interface. Also because I want to put the scripts in the sequence I used them its more convenient to put the second python script in the GIPSY section.

### A.1 Changing the radius from arcseconds to parsecs for variable distances

The aim of this program is to read a datafile, containing the observed rotation curve data, which consists of 3 columns. The first column contains the radius in arcseconds. ROTMAS, the rotation curve fit program of GIPSY, requires the radius in parsecs. Thus the data for this first column has to be changed from units of arcseconds to units of parsecs. However, because the distance is used as a free parameter, the radius of the observed rotation curve will also change for each different distance.

After reading the datafile a range of files is created for a set distance range. Because python does not include the last number, you have to set it +1 to have the full range. After creating the new datafile, the first column of the original file is read and recalculated to units of parsecs, after which it is written to the first column of the new datafile. The second and third column, containing the velocities remain the same. Because integer values are used the distances are set in  $10^{-1}$  Mpc.

```
#!/usr/bin/python

from Numeric import *
import math

file1 = 'u128rc.dat'

def readfile(file):
    a = open(file1,'r').readlines()
    a = filter(lambda x:x[0]!='#',a)
    a = map(string.split,a)
    return a

e=readfile(file1)

for d in range(530,571):          #2nd number not included in the files, so +1
    file2 = 'u128rc%s.dat' % d
    f = open(file2,'w')
    for i in range(len(e)):
        f.write(str((float(e[i][0])*(float(d)/10)*0.290888/60)))
        f.write(' ')
        f.write(e[i][1])
        f.write(' ')
        f.write(e[i][2])
        f.write('\n')
    f.close()
```

## B GIPSY programs

GIPSY (Groningen Image Processing System) has been developed at the Kapteyn Institute to make fits of observed rotation curves. In this package a MOND-fit is also included with the possibility to use the gas, the disk and the bulge contribution as free parameters and also  $a_0$ . The package did not include the possibility to use the distance as a free parameter, thus it was necessary to write additional algorithms to make this possible. GIPSY has been written in a very flexible way. All input data can easily be automated with simple scripts, using python or perl-scripts, a coding language specifically made for GIPSY. The easiness with which this could be done greatly enhanced the speed with which could be proceeded to do the fits.

### B.1 Variable Distances

GIPSY has a manual input to convert the measured data for the gas and stellar contribution into units of kpc and  $M_\odot \text{ pc}^{-2}$  with an output in a new datafile which is needed so you can use it in ROTMAS. This function is called ROTMOD. Among the input parameters is also the distance to the galaxy and hence the place where to automate the conversion for a range of distances. The program was named vardis.col. Two for-loops were made: one for the disk contribution (intensity) and one for the gas contribution.

The input distances are in  $10^{-1}$  Mpc because of the use of integer values. Radii are in kpc with steps of 1 kpc. Disk masses are in  $10^9 M_\odot$ . If the mass is based on the luminosity with a ratio of 1, the fitting of the rotation curve will use the stellar mass-to-light ratio. Using a mass not based on this means fitting will be done using the adopted mass that has been put in ROTMOD. The mass-to-light ratio will have to be calculated with the new stellar mass found, divided by the luminosity that has been corrected for the new distance. Running the script happens with just typing vardis in the GIPSY interface.

```
!Variable Distance
integer var
```

```
for var=530,570
"ROTMOD
DENSITY= file(u128im.rcl,2,:) ; DISTANCE=%var/10 FILE= u128i%var.kod
GGIOPT= FIXCOL MASS= 5.7 PAIRS= n RADII= 0:40:1 RADIUS= file(u128im.rcl,1,:) ;
TYPE= disk UNITS= arcsec,mag/arcsec**2 USER= y ZLAW="
cfor
```

```
for var=530,570
"ROTMOD
DENSITY= file(u128gd.rcl,2,:) ; DISTANCE=%var/10 FILE= u128gd%var.kod
GGIOPT= FIXCOL MASS= PAIRS= n RADII= 0:40:1 RADIUS= file(u128gd.rcl,1,:) ;
TYPE= disk UNITS= arcsec,msun/pc**2 USER= y ZLAW="
cfor
```

## B.2 Data File Changer

ROTMAS makes use of a graphical interface, but the inputdata can easily be changed from the GIPSY terminal. Changing the inputfiles for a different distance would require several inputs to be changed. Thus for this part to be able to do a quick change of distance a script was written in python which was called DFC (Data File Changer). It does not require an extension and this way it could be run in GIPSY just by typing `dfc` in the terminal.

DFC changes the inputparameters for all the necessary inputdata needed for ROTMAS. The BOX addition is to make it rescale the curve graph to the new dataset when changing to a different distance. The same is the case for the gas mass (MG) which has to be rescaled to its fixed value when changing datafile. When you change to a different galaxy you will need to clear the axis box in ROTMAS and press <enter> to rescale, after which DFC works normally again. ROTMAS might give an error, but everything will work normally afterwards. With DFC there is no need to put in the data when starting ROTMAS. Just running DFC and giving the first distance will fill in all the required fields, after which you only need to activate the MOND fit option in ROTMAS.

The input in GIPSY when running DFC is  $10^{-1}$  Mpc, because of the way `vardis` can only generate the different distance datafiles. The addition of ROTMAS in the `wkey` is to be able to change the inputdata in ROTMAS and not somewhere else or nowhere at all. The different inputparameters can be retrieved by typing `!ROTMAS` when starting ROTMAS. This will show the effects of what you do in ROTMAS in the GIPSY interface.

```
#Data File Changer
#!/usr/bin/env python
from gipsy import *

init()

while True:
    df = userint('DATAFILE=', 'Give a filename', default=1, defval=-1)
    cancel('DATAFILE=')
    if df==-1:
        finis()
    wkey('FILE=u128rc%d.dat' %df, 'ROTMAS')
    wkey('ROWS= :', 'ROTMAS')
    wkey('RADII_COL= 1', 'ROTMAS')
    wkey('ODATA_COL= 2', 'ROTMAS')
    wkey('ERRORS_COL= 3', 'ROTMAS')
    wkey('RADII=file(u128rc%d.dat,1,:)' %df, 'ROTMAS')
    wkey('ODATA=file(u128rc%d.dat,2,:)' %df, 'ROTMAS')
    wkey('ERRORS=file(u128rc%d.dat,3,:)' %df, 'ROTMAS')
    wkey('GRADII=file(u128gd%d.kod,1,:)' %df, 'ROTMAS')
    wkey('GDATA=file(u128gd%d.kod,3,:)' %df, 'ROTMAS')
    wkey('DRADII=file(u128i%d.kod,1,:)' %df, 'ROTMAS')
    wkey('DDATA=file(u128i%d.kod,3,:)' %df, 'ROTMAS')
    wkey('MG= 1.3', 'ROTMAS')
    wkey('MG_FIX= YES', 'ROTMAS')
    wkey('A= 4055.0000', 'ROTMAS')
    wkey('A_FIX= YES', 'ROTMAS')
    wkey('BOX=;', 'ROTMAS')
```

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